

Exercises with a *Mathematics* Dictionary ~ 2

Students need help and encouragement in familiarising themselves with any work of reference if they are to make the most of it. The purpose of these pages is to provide a framework that will allow them to become familiar with a particular *Mathematics* Dictionary. For details of the dictionary for which these exercises are intended, see the note at the bottom.

Broadly speaking there are three stages in looking up any piece of information. These are: finding, understanding and using. The last two are separate but tend to be fused together since the easiest way of checking that a new idea has been understood is by asking for it to be used. The familiarisation exercises suggested here are presented as seven worksheets which can be photocopied.

Students meeting the intended dictionary for the first time will undoubtedly want some help in finding their way via the Wordfinder, but this probably need be no more than a few oral exercises.

The worksheets are graduated and the sheet most appropriate for an individual, group, or class will be for teachers to decide. Each sheet contains 25 questions.

Set E is concerned almost solely with finding information and copying out the appropriate number or word. There is no requirement for any (mathematical) understanding. For most of the questions, the page number is given as well as the word to be looked for. Only for the last 6 will the Wordfinder be needed.

Set F is concerned with ‘finding’, and contains plenty of help, but now some ‘using’ has to be done also. For all questions the keyword which it is necessary to find in order to answer the question is printed in **bold**, and over half the questions also give the necessary page numbers. No calculations are required.

Set G gives a little less help with ‘finding’, but every keyword is printed in **bold**. There is now more ‘using’ to be done. A basic calculator is adequate for the computation needed.

Set H does not give any help with page numbers and only half the keywords are printed in bold. A basic calculator will suffice for nearly all questions.

Set I At this level students are expected to identify most of the keywords for themselves or else try various possibilities. A scientific calculator is necessary for several questions.

Set J involves the finding and using of a variety of formulas. A calculator is needed.

Set K gives no help in ‘finding’ and the questions are more demanding.

How this work is used will depend on circumstances. If there is a class-set of dictionaries then the whole class could be engaged in the work at one time. Given that only a few dictionaries are available then it would be possible to work with one group at a time provided that the class organisation allowed this. Even if these exercises are not used in the classroom, individuals with their own copy of the dictionary might welcome the opportunity to do some structured work with it.

The dictionary might also be used as a starting-point for a piece of work. Some possibilities for this are given here on pages 9 and 10 as *Matters arising* . . .

These exercises were compiled for use with the *Oxford Mathematics Study Dictionary*, 2nd edition 1999 ISBN 0-19-914567-9. These exercises **do not** fit the 1996 first edition for which separate sets of exercises are available. The sheets provided here have been started with Set E to avoid confusion with the earlier sets (A to D) for the 1st edition.

See the last page for more details about this dictionary.

Dictionary familiarisation exercises - Set E

Help is given by printing in **bold** the word under which the information is to be found. In most cases the necessary page number is also given. If it is not, then the word(s) will have to be looked up in the Wordfinder.

1. [page 2] What does **UT** stand for?
2. [page 15] What is the **digital root** of 8579?
3. [page 56] How many bytes are there in 1 **Gb**?
4. [page 68] In which year was **Pythagoras** born?
5. [page 98] Give another name for the shape known as an **arrowhead**.
6. [page 129] Who introduced the **=** sign, and in which year did he do so?
7. [page 160] Give the Greek and Latin words for a **hundred**.

Answers to the next 12 questions will be found in the appropriate table on the page listed.

8. [page 77] In how many ways is it possible to **partition** the number 7?
9. [page 19] What was the value of the **Retail Prices Index** in 1998?
10. [page 103] How many different **polyominoes** can be made from 8 squares?
11. [page 163] How many **derangements** ($!n$) are possible when $n = 5$?
12. [page 45] For the number 26, list all its **factors**.
13. [page 88] What is the value of **$P_e(n)$** when $e = 5$ and $n = 6$?
14. [page 150] How many milligrams are there in a **mass** of 1 carat?
15. [page 128] What is the value of **$\pi(1000)$** ?
16. [page 103] What SI **prefix** does the letter P stand for, and by how much must the unit be multiplied?
17. [page 166] Give the 6th **pyramid number** for a triangular pyramid.
18. [page 79] Find the value of **nC_r** when $n = 10$ and $r = 7$
19. [page 151] By what should a distance in yards be multiplied to change it into metres?

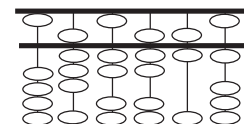
In each of the next 6 questions, the word printed in **bold** will have to be found by using the Wordfinder.

20. What is the value of the **persistence** of 79?
21. Give the **median** value of 6, 2, 12, 4, 7, 18
22. Give the value of the **golden ratio**.
23. How many metres are there in a **nautical mile**?
24. List those subjects which were studied for the **quadrivium**.
25. Give the Latin and Greek **prefixes** for 6.

Dictionary familiarisation exercises - Set F

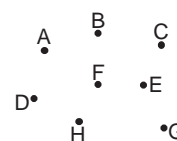
Help is given by printing in **bold** the word under which the information is to be found. In some cases the page number is also given. If it is not, then the word(s) will have to be looked up in the Wordfinder.

1. [page 12] Write down the size of any angle, in degrees, which is **obtuse**.
2. [page 14] Write down a group of five **consecutive numbers** starting with 11
3. [page 17] What is the value of a **discount** of 10% on £12 ?
4. [page 21] Which number is the **divisor** in the sum $798 \div 14 = 57$?
5. [page 22] What is the number showing in the **soroban** drawn on the right?



6. [pages 38/39] Draw a **simple closed curve**.
7. [pages 40/41] How many cusps does a **cardioid** have ?
8. [page 43] Write down the next line of **Pascal's triangle** after the one given.
9. [page 44] Write down the **proper divisors** of 12
10. [page 92] What is the name of a **regular convex polyhedron** having 20 faces?
11. [page 50] Which number is the **numerator** in the fraction $\frac{7}{10}$?
12. [pages 70] Is it possible that **Mersenne** and **Fermat** could have known one another? Explain why.

13. [page 52] Write down the letters of the 3 dots which are **collinear** in the drawing on the right.



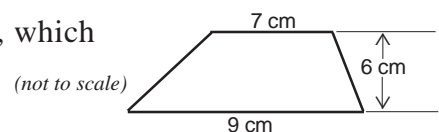
14. [page 81] What is the total value of these symbols written in the **early Roman number system**?
MCCLXXVIII

15. Give the **angle sum** of a quadrilateral.
16. How many faces does a **dodecahedron** have?
17. In the **bar chart** shown [on page 117], what was the shoe size found most often?
18. Which of these letters are **asymmetric**? A E F G J K L O Q U X Z
19. Use the temperature **conversion scale** to change 140°F into °C.
20. In an **isosceles triangle** how many edges are of the same length?
21. What **conversion factor** would you need to change gallons(UK) into litres?
22. Write down a number which is a **palindrome**.
23. Write out the value of a **trillion** in full.
24. Give another name for a **cube**.
25. How many lines of symmetry does the shape known as a **kite** have?

Help is given by printing in **bold** the word under which the information is to be found. In some cases the necessary page number is also given. If it is not, then the word(s) will have to be looked up in the Wordfinder.

A calculator may be used. Where needed, use a value for π of 3.14.

1. [page 12] Write down the size of any angle, in degrees, which is **reflex**.
2. [page 15] Give the **digit sum** of 8025.
3. [page 26] Find the area of a **circle** which has a radius of 4 cm.
4. [page 36] A point is marked on a grid at (6,4). What is the value of the **abscissa**?
5. [page 40] What is the length of one arch of a **cycloid** when $a = 3$ cm?
6. [page 44] List the **proper factors** of 18.
7. [page 50] Write down an **improper fraction**.
8. [page 56] How many bytes are there in 6**Kb**?
9. [page 74] What is the **compass angle** of the direction described as East?
10. [page 77] What is the value of the **persistence** of the number 46 ?
11. What is the largest symbol used in the **hexadecimal** system?
12. Find the **angle sum** of a polygon having 5 edges.
13. Find the volume of a pyramid which has a **perpendicular height** of 12 cm and an area of base = 10 cm².
14. What is the total of the top row of the **magic square** given as an example of using primes only?
15. Use a **two-way table** for combining the throws of 2 dice, to find in how many ways a total of 6 can be made.
16. In the **pictogram** shown on page 117, how many pupils in Class 2 owned a computer?
17. Find the **arithmetic mean** of 7, 3, 2, 9, 11, and 10
18. In the **box and whisker diagram** what is the value of the median being shown in the example diagram?
19. Use a flow diagram to change a temperature of 92.3° on the **Fahrenheit scale** to a temperature on the Celsius scale.
20. What **conversion factor** would you use to change yards into metres?
Use it to change 8 yards into metres.
21. Which **prefix** (for SI units) means “multiply by 1 000 000 000”?
22. List all the **prime numbers** between 80 and 100
23. What is the **complement** of an angle of 34° ?
24. Use a **formula** to find the area of the trapezium shown, which has parallel edges of length 9 and 7 cm, with a perpendicular height of 6 cm.
25. One angle in a pair of **supplementary angles** is 27°.
What is the size of the other angle?



Dictionary familiarisation exercises - Set H

Help is given in some cases by printing in **bold** the word(s) under which the required information being asked for is to be found.

A calculator may be used. Give all answers to a suitable degree of accuracy.

1. Write down the **transpose** of this matrix $\begin{pmatrix} -3 & 5 \\ 7 & 1 \end{pmatrix}$
2. Which is bigger: a **googol** or a **centillion** ?
3. Find the area of a **sector** of a circle which has a radius of 7 cm and an angle at the centre of 50 degrees.
4. In the **histogram** given as an example, estimate the number of people in the 40 to 50 age group.
5. Give the value of -90°C as a temperature on the **Kelvin scale**.
6. What is the area enclosed by a **cardioid** when $a = 3$ cm?
7. The bearing of A from B is 217° . Give the **reciprocal bearing** of B from A.
8. Find the area of a regular **decagon** which has an edge length of 5 cm.
9. What is the **supplement** of an angle of 75 degrees?
10. Use **reverse percentages** to work out the original value of a camera which has had its price increased by 20% and is now marked at £102
11. Give the **digital root** of the number 16437
12. Work out the volume of a **frustum of a pyramid** when $A = 20 \text{ cm}^2$ $B = 45 \text{ cm}^2$ and the distance between the faces is 10cm.
13. Give the value of the 11th term in the **Lucas sequence**.
14. Find the area of an ellipse having the values $a = 8$ cm and $b = 10$ cm.
15. Convert 40 gallons(UK) into its equivalent in litres.
16. Work out the value of the Mersenne prime for the case when $n = 5$
17. Find a solution to the asterithm given as an example and write out the sum in full.
18. How many lines of symmetry does a hypocycloid have when $n = 4$?
19. Show that 2196 is a Harshad number.
20. From the combination table for nC_r , read off how many ways there are of choosing 6 objects from 11
21. Calculate the area of curved surface of a cylinder having a height of 12 cm and a radius of 4 cm.
22. Name two quadrilaterals which can NEVER be cyclic.
23. In the stem and leaf plot shown as an example, what is the frequency of the data having a stem value of 2 ?
24. Change the polar coordinates $(7.6, 58^{\circ})$ into Cartesian coordinates.
25. How many counters are needed to make a polygon number in the shape of a hexagon having eight counters along each edge?

Dictionary familiarisation exercises - Set I

Help is given, in some cases, by printing in **bold** the word(s) under which the required information is to be found.

A calculator may be used. Give all answers to a suitable degree of accuracy.

1. Write down two angles which form a **conjugate** pair.
2. Show, by **casting out 9's**, that $258 \times 47 = 12156$ is wrong.
3. Make a sketch of the cross-section of a square **antiprism** when a cut is made parallel to one of the square end faces.
4. Give the value of the **Merenne prime** when $n = 7$
5. Rework the example given under **iteration** and find the value of x_5 to 6 decimal places.
6. An oblong **rep-tile** is to be made having one edge of length 8cm. What must be the length of its other edge?
7. Find a solution to the **alphametic** given and write down the values of all the letters.
Hint: E is 5; N is 6
8. Solve the **Diophantine problem** given as an example.
9. From all the approximations given for **π** on page 86, which is closest to the correct value?
10. Write the 11th and 12th terms of the **Lucas sequence**.
11. Give the set which is the symmetric difference of $\{3, 6, 7, 10\}$ and $\{5, 6, 7, 8\}$
12. A triangle has an area of 60cm^2 . What is the area of its median triangle?
13. Convert 40 gallons(UK) into its equivalent in gallons(US).
14. How many times bigger is a petametre than a picometre?
15. Work out the value of the trace of this matrix

$$\begin{pmatrix} 2 & -4 & 1 \\ 0 & -9 & 5 \\ 8 & 3 & 6 \end{pmatrix}$$

16. Find the geometric mean of $\{3, 5, 7, 9\}$
17. List all the aliquot parts of 20
18. Show that Goldbach's conjecture works for the number 24
19. Which is bigger - a googol or a centillion - and how many times bigger is it?
20. A regular octagon has an edge length of 4cm. What is its area?
21. A regular dodecahedron has a volume of 600 cm^3 . What is the length of one edge?
22. Show that, for the cubic equation used as an example under 'trial and improvement', 0.89 is a better value for the root than the one given.
23. In a circle there are two intersecting chords, MN and PQ, which cross at X.
MX is 10cm, NX is 7 cm and PX is 5cm. What is the length of PQ?
24. Sketch an Argand diagram which goes from 0 to 5 on both axes and on it mark the positions of the three points A, B and C which represent these numbers. Label each point with the appropriate letter.

A is $3 + 4i$
B is $\sqrt{-1}$
C is $\sqrt{-17}$
25. A cylinder has height of 20 cm. A piece of string 30 cm long is wound once around the cylinder to form a helix. The string starts at the bottom of the cylinder and finishes at the top, at a point exactly above where it started. What is the diameter of the cylinder?

Dictionary familiarisation exercises - Set J

Answers to all of these problems can be worked out by using the appropriate formula to be found under the correct keyword, and these are printed in **bold**. The relevant [page number] is also given in some questions. Use $\pi = 3.14$ where needed and give all answers to a suitable degree of accuracy.

1. [16] Find the (new) value of a car, whose original value was £9000, after **depreciation** of 20% per year for a period of 5 years.
2. [18] A computer is advertised at £1045.75 (inc. VAT). Use the **reverse percentage** technique to calculate its original cost. The rate of VAT is 17.5%
3. [26] Find the area of a **sector** of a circle when the angle at the centre is 70° and the area of the full circle is 56 cm^2
4. [31] Calculate the distance AB between the centres of two **orthogonal circles** having $R = 8 \text{ cm}$ and $r = 6 \text{ cm}$.
5. [32] What is the area of the **curved surface of a cone** which has a base radius of 6.5 cm and a slant height of 11 cm?
6. [40] Find the perimeter and area of an **astroid** when $a = 7.5 \text{ cm}$.
7. [43] Give the value of the **Pythagorean triplet** made when $m = 7$ and $n = 4$
8. [74] If the bearing from A to B is 260° , what is the **reciprocal bearing**?
9. Calculate the sum of an **arithmetic series** given that : $a = 4$, $d = 3$, $n = 20$
10. Find the approximate volume of a **barrel** which has an end diameter of 40cm, a middle diameter of 60cm, and a height of 90cm.
11. What is the value of the **centred-polygon number, $C_e(n)$** when $e = 5$, $n = 4$?
12. Find the value of the **permutation, " P_r "** when, from 10 objects, 7 are chosen at a time.
13. Give the volume of a circular **paraboloid** when $d = 5\text{cm}$, and $h = 8\text{cm}$.
14. Use **Heron's formula** to find the area of a triangle given $a = 5\text{cm}$, $b = 7\text{cm}$, $c = 10\text{cm}$.
15. Use **Ramanujan's formula** to find the perimeter of an ellipse when $a = 8 \text{ cm}$ and $b = 10 \text{ cm}$.
16. Work out the **instalments** to be paid at monthly intervals on a loan of £500 for 1 year at a rate of interest of 2.5% compounded monthly.
17. Change the **polar coordinates** $(4, 35^\circ)$ into Cartesian coordinates.
18. Find the area of a **segment** of a circle when $\theta = 40^\circ$ and $r = 7 \text{ cm}$.
19. What is the value of **haversine** 40° ?
20. How many balls (or similar objects) would be needed to make up a **pyramid number** built on a triangular base and 10 layers high?
21. In the **Fibonacci sequence** find the value of F_{20}
22. Calculate the area of an **ellipse** whose major axis is 9 cm and minor axis is 7 cm.
23. Use the **cosine rule** to find the size of angle A when $a = 10\text{cm}$, $b = 16\text{cm}$, and $c = 21\text{cm}$.
24. Find the **area** of a triangle which has $a = 7.3\text{cm}$, $c = 10.8\text{cm}$, and $B = 40^\circ$
25. Use the **sine rule** to find the length of a when $b = 8.2\text{cm}$ $A = 55^\circ$ and $B = 65^\circ$

Dictionary familiarisation exercises - Set K

1. In January 1975 the RPI was 119.9 and rose to 147.9 by the following January. What was the rate of inflation in that period?
2. A sector of area 15 cm^2 is to be cut from a circle of radius 6.8 cm. What is the angle at the centre of the sector?
3. From a point O outside a circle a secant is drawn cutting the circle at points A and B with $OA = 3 \text{ cm}$ and $AB = 5 \text{ cm}$. What is the length of the tangent to the circle from O to the point of contact?
4. Calculate the radius of the circumcircle to the triangle ABC when $a = 7.8 \text{ cm}$, $b = 5.4 \text{ cm}$, $c = 10.3 \text{ cm}$.
5. On page 39 calculate the length of the locus drawn as the string unwinds. Assume the block to measure 5 by 10 mm.
6. A nephroid has an area of 400 cm^2 . What is the length of its perimeter?
7. Use Euclid's algorithm to find the largest common factor of 89693 and 265589.
8. In the diagram of a cross-staff, what would be the subtended angle when the cross-bar ($AB = 20 \text{ cm}$) was 34 cm from E?
9. Work out the value of the combination given by ${}^{20}C_{15}$
10. Determine whether 70 is a happy number or not.
11. What is the percentage error in the value for π given by the mean of the two values found by Archimedes?
12. A 300 mm sine bar is to be set at an angle of 27.5 degrees. What must be the height difference between the two supporting blocks?
13. What will be the greatest edge-length possible when a dodecahedron is made by cutting faces on a sphere of diameter 15 cm?
14. Show that the formula for the volume of a barrel is correct for a cylinder.
15. Find the persistence of the number 868
16. Find the sum of the first 20 terms of a geometric series having a first term of 3, and a constant multiplier of 2.
17. Give the conversion factor for changing gallons/foot² to litres/metre².
18. Express the decimal fraction 0.875 as a bicimal.
19. Write out all the possible derangements of the ordered set of letters {A, B, C, D}
20. Find the surface area of a torus having a circular cross-section 5 cm in diameter and a volume of 100 cm^3 .
21. Give an approximate value for $\cos 30^\circ$ obtained by using only the first 3 terms of the series for that trigonometric function.
22. Evaluate this expression written in reverse Polish notation: $4 \ 2 \ 7 \ + \ \times \ 5 \ 3 \ \times \ 9 \ + \ \times$
23. How many barrels (*the measure*) are there in 1 cubic metre?
24. Depreciation on one particular car is assumed to run at 15% annually. This puts its value after 8 years at £4272. What was its original value?
25. Re-write this expression using postfix notation: $(3(1 + 2) \times 4(5 + 6) + 7) + 8$

Many entries can be used as discussion points, set pieces of work, specific investigations or open-ended enquiries. On this page and the next are suggestions for generating ideas for further work.

Devise a **mnemonic** to remember the square root of 3; or to recall as many digits of π as you can; or to remember the order of the SI prefixes - perhaps one for those greater than 1 and another for those less than 1.

Construct a **direct proof** that adding an even and an odd number makes an odd number.

Construct a **visual proof** that adding an even and an odd number makes an odd number.

Do both of the preceding for subtraction or multiplication of, say, odd numbers and even numbers. Or odd and odd.

A **square** is defined as “a rectangle whose edges are all the same length”. A direct result of that is “*It has four lines of symmetry*”. Would it be possible to define a square as “a quadrilateral having four lines of symmetry”? Would it be desirable or more useful? Find other definitions which can be turned around - and make sure they are true!

Investigate the **digital roots** of square numbers - or any other types of numbers.

An explanation is given of how to change **polar coordinates** into Cartesian coordinates. Explain how the reverse can be done.

Design and make a **conversion scale** for changing litres into gallons; or a *Shoppers Guide* for changing price per kilogram into price per pound.

Investigate the **locus** shown on page 39. The rectangle is 5 mm by 10 mm.

- What is the length of the curve drawn in red?
- What would be its length for a rectangle x mm by y mm?
- Study how the area being swept (*in 90° steps*) by the string varies as it unwinds .

Almost every term explained under **recreational mathematics** could serve as the basis of either a single task or a major topic. For example:

- Make a set of **pentominoes** (cut out of card) and assemble them to make a complete rectangle.
- Find the 12 different **hexominoes** and assemble them into various shapes.
- Make a set of **Soma cubes** and put them together to make a cube.

The dictionary gives a lot of basic information, but much more can be obtained by building upon what is given. For instance:

- Make a formula or flow diagram to change temperatures on the Réaumur scale (p.136) into their equivalent on the Celsius scale.
- Use the **sine curve**, and a scientific calculator, to find the value of $\sin 2000^\circ$ or some other value which is outside the range of your calculator.
- From the information given on p.56, deduce how many bytes there are in a Terabyte.
- Try to decide, and then describe, how the **suan pan** works for the simple case of adding two numbers. Then determine how the **soroban** manages the same task in spite of having two beads less on each wire. A simple model can be made by drawing suitable lines on a piece of paper and using counters to represent the beads.
- Design and make a **slide rule** (from two pieces of card) to multiply together any two whole numbers from 1 to 9. The calibration of these scales will have to be done by trial and error.

Compare the **decomposition** and **equal addition** methods of subtraction. Is one 'easier' than the other? Does the answer to that question depend upon what the numbers are? For instance, try both methods on $703 - 486$.

Check the **nomogram** illustrated by doing some calculations and seeing how close the calculated values are to those given by the nomogram. Does it vary with the position you are using on the scale?

Investigate how you might find the size of an **ellipse** that was required to have a given area, or a given perimeter length, or both.

The diagram for **Eratosthenes' sieve** shows some lines have more prime numbers on them than others. Some have none at all. Look at other ways of setting the numbers out and try to find one that lines up the maximum number of primes.

Investigate the relationships between **polygon numbers**

(a) of the same type (b) of different types (c) of some other (invented) types.

Find out about the **odds** offered in horse-racing. For one particular race, change all the odds offered into probabilities and add them up. What do you notice? Does it always happen?

π With only a limited knowledge of trigonometry of the right-angled triangle, Archimedes' method for approximating π (p.86) by 'squeezing' a circle between two polygons can be followed through for a hexagon, or some other polygon with a greater number of edges. The algorithm (p.87) can be worked through with a calculator for two passes, though the second pass will not be completely accurate. It is a valuable exercise in recording the different values as they change.

Most of the terms given under **number diversions** suggest starting points for further investigation. Some could be done by hand, and nearly all of them represent an obvious challenge for those capable of writing computer programs. There are many other opportunities for work on a computer.

On page 41 two epicycloids are shown, each having 5 cusps but, they have different n values. Why does this happen? Investigate the number of cusps an **epicycloid** (or a **hypocycloid**) will have when n is not a whole number. This is probably best done by writing a computer program for generating them. This can then be extended by attempting to draw the locus of a point which is NOT ON the circumference of the rolling circle but either inside or outside it. This creates the trochoids.

Most calculators will not give an exact value for the **factorial** of any number beyond that for 11. Produce a computer program that gives exact values up to $30!$ (or more).

Write a computer program to:

- produce a **proof by exhaustion** that "between every pair of square numbers less than X there is at least one prime number". Start with a small value of X and then see how large you can make it.

How can you demonstrate that your search is correct?

- produce **abundant** and **deficient numbers**. Which are there most of? Find an abundant number which is odd.
- draw a **spiral**.
- find the **persistence** of any number.
- evaluate the number of steps needed to produce **Kaprekar's constant** from any given four digits. Then investigate if there is a similar effect when three, or five, or six, digits are used
- evaluate **π** to (say) 1000 decimal places by use of Shank's method as outlined at the top of page 87.

12 sample pages from the
The Oxford Mathematics Study Dictionary
2nd edition
can be viewed at this site.

From the CIMT Home Page
<http://www.ex.ac.uk/cimt/>
select

Mathematical Topics
Then, from that page, select
Dictionary